DOCUMENT RESUME

ED 299 102 SE 049 462

TITLE A Cognitive Taxonomy for Mathematics Teaching. School Research Newsletter 1988:4.
INSTITUTION National Swedish Board of Education, Stockholm.
PUB DATE Jun 88
NOTE 11p.
AVAILABLE FROM Swedish National Board of Education, Department for Coordination and Planning, S-106 42 Stockholm (free while supply lasts).
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Classification; Cognitive Development; *Concept Formation; Curriculum Development; Elementary Education; *Elementary School Mathematics; Foreign Countries; *Holistic Approach; *Learning Theories; Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; Mathematics Materials

ABSTRACT

This document outlines a theory of learning and education for the teaching of mathematics. The theory is termed a cognitive taxonomy. The taxonomy, developed over a 15-year period, is based on the connection between mathematical material and pupils' reactions to it. The function of this connection is termed a process. The taxonomy is based on the idea that the planning of mathematics teaching should take into account both pupils and subject matter. Research should elucidate and interpret, on a parallel basis, the way in which the subject matter can be arranged for pupils and the way in which individual pupils react to the subject matter. (PK)

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SCHOOL RESEARCH NEWSLETTER

JUNE 1988 FIELD-TOPIC Mathematics teaching 1988:4

Current project:

TITLE OF PROJECT: A COGNITIVE TAXONOMY FOR MATHEMATICS TEACHING

This project has been in progress for fifteen years. Its aim is to devise methods of studying pupils' mathematical learning and to construct a theoretical basis for the teaching of mathematics. The project team have developed a theory of learning and education for the elementary teaching of mathematics. This teory they therm a cognitive taxonomy.

The taxonomy is constructed in accordance with the task process model by Uprichard et al. (1975), supplemented with the project team's cognitive theory of learning and education. This taxonomy is based on the connection between mathematical material and pupils' reactions to it. The function of this connection is termed a process. The project team are trying to take a holistic view of personality. In their opinion, the planning of mathematics teaching should take into account both pupils and subject matter. Research should elucidate and interpret, on a parallel basis, the way in which the subject matter can be arranged for the pupils and the way in which the individual pupils react to the subject matter.

PROJECT IN PROGRESS AT:
Malmö School of Education
Lund University
Department of Education
Box 23501
S-200 45 MALMÖ
Tel: 101/ 32 02 46

SCIENTIFIC LEADER:
Olof Magne

PROJECT LEADER:
Olof Magne

PROJECT ASSISTANT:
Kerstin Thörn

U.S. DEPARTMENT OF EDUCATION
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SWEDISH NATIONAL BOARD OF EDUCATION

NEWSLETTER 1983 - 1988

1983:1 Evaluation of Courses of Study at Upper Secondary School (UTGY)
1983:2 The Research Programme of The National Swedish Board of Education
1983:3 Science Subjects - An Evaluation of Student Achivement and Attitudes in International Perspective (IEA Science II)
1983:4 Evaluation through follow-up of students (UGU)
1983:5 Analysis of teacher-directed experimentation

1984:1 Children with early perceptual funcional disturbances. A follow-up during school years
1984:2 The SIAu Project: Project description
1984:3 Transition from compulsory and upper secondary school to further education or employment - the GUARANTEE project
1984:4 Vertical integration at junior and intermediate levels
1984:5 Learning to read
1984:6 Family support and development - a longitudinal, ecological study of families with children of fre-school and junior-level age (the FAST-Project)
1984:7 Study assistance and adult study assistance - utilisation and effects
1984:8 UTRES - Evaluation of resource utilisation in upper secondary schools
1984:9 MEDIA PANEL

1985:1 Creativity and communication: music and art education - The KROK-project
1985:2 Childrens participation at junior level
1985:3 Programme for recurrent educational situation reports: school description models
1985:4 Discovery learning and cognitive development - the UKU-project
1985:5 Man in a changing society - MIS
1985:6 Systematic evaluation of the "forest in schools" project
1985:7 Evaluation of experimental concentration of studies in upper secondary school
1985:8 Pupils´working environment at school in use (EMILIA)
1985:9 Immigrant pupils at the senior level of compulsory school and in the upper secondary school

1986:1 Education - Society; Curricular theory
1986:2 IEA Written composition study
1986:3 UTRES - Evaluation of resource utilisation in upper secondary school
1986:4 Technology and science in school and society: Education, training, subsequent training
1986:5 Historical image and historical awareness
1986:6 Political and citizenship education in Swedish schools in the 20th century (the role and concept formation of social subjects)
1986:7 Feasibility of joint studies in adult education
1986:8 Studies of the innovation process in compulsory school
1986:9 Girls in science and technology
1986:10 Evaluation trough pupil follow-up - the UGU project

1987:1 Young people and peace - prerequisites for peaceoriented istruction
1987:2 Functional opportunities of the school administration in municipalities with local agencies and free municipality experiments
1987:3 Steering strategies in a decentralised decitionmaking system
1987:4 The outward efficiency of schools
1987:5 Swedish in upper secondary school - two worlds?

1988:1 META STUDIES
1988:2 Pupils´ frames of reference and the content of teaching
1988:3 Follow-up of the in-service training programme "An upper secondary school for all"

Magne-Thörn's cognitive taxonomy

Mathematics teaching has developed sporadically. Magne (1986) has pointed out that Sweden in the 1950s was exploring new lines (e.g. Ferner, Hultman, Lindström, Magne, Malmer, Sjöholm, Vanäs, Werdelin, Wigfors). Unfortunately this promising trend was broken by the "new math" and its Americanisms. "Back to basics" propaganda then turned the clock back in certain respects to the traditions of an earlier period. Perhaps we are still there.

Both research and teaching are in need of redirection. Leading ideas should have a holistic emphasis. There must be better balance between different factors of subject matter and teaching. Pupils' reactions to subject matter should be taken more adequately into account and made part of a system.

This view is propounded in a survey report by Olof Magne and Kerstin Thörn. Magne and Thörn recently published a report entitled "A cognitive taxonomy for mathematics teaching", Pedagogiska problem (Malmö: Lärarhögskolan), No. 471-472, 1987 (Address: Box 23501, S-200 45 MALMÖ).

Taxonomy means division or classification, and cognition refers to knowing or thinking. We are thus concerned with analyses of mathematical processes of thought. The authors have scrutinised the way in which children work with mathematics, both correctly and incorrectly, in more than a million operations. The analyses of errors gave rise to the system of categories presented by the authors in their report. The intention is for teachers to have less difficulty in planning teaching and guidance. Olof Magne and Kerstin Thörn devote special thought to pupils who are unsuccessful, those who make a lot of mistakes in mathematics.

The category system

The category system operates on three levels.

(1) The authors gradually arrived at six main fields. The most general level comprises educational goals for the teaching of mathematics in compulsory school. Those educational goals refer to such things as

subject matter balance and allowance for the pupils' development. The main fields are:

Field P: Language comprehension and problems.
Field T: Speech comprehension.
Field G: Form perception, money, geometry, measurement, units.
Field ASMD: The four rules of arithmetic.
Field F: Functions, equations, algebra.
Field B: Descriptive statistics, probabilities.

The last two mentioned fields have not been treated.

(2) The categories comprise observations of the pupils' reactions to the mathematical material, i.e. their thinking processes and motor processes. The investigation involved given the pupils a variety of mathematical problems. Wide variety was needed, and the study had to include much more besides numerical calculation, e.g. perceptual, motor and linguistic actions. The authors noted correct and incorrect reactions, entering them in the category system.

Pupil reactions to the subject matter were termed categories. The categories are thus included as parts of the main fields.

(3) Specific pupil reactions. The authors frequently constructed subheadings for the categories. These can be interpreted as specific pupil reactions. They have not been fully investigated; this will be a task for the future. The authors believe it is possible to construct hierarchies within the categories. Thus, in their opinion, it is the specific pupil reactions which can be included in hierarchies.

A developmental view of the main fields

Mathematical processes begin long before children start school. This is seldom presupposed in the curricula, but it is also interesting to note that the commencement of instruction varies from one country to another. In Sweden, mathematics is supposed to be learned from age 7 onwards. In England, mathematics teaching begins at age 5. Mathematics ought to be taught in pre-school education. We must start with the newborn period.

Three main fields - P, T and G - appear to contain fundamental or original elements of this kind.

P: Language comprehension, e.g. in mathematics and problem solving, is based on language. Developmental psychology teaches us that children develop their reasoning capacity with the aid of language, even though this is not the sole prerequisite.

Uncertain language comprehension, e.g. regarding mathematical processes, and lack of capacity for collecting, interpreting and using information about problems can be the cause of errors in problem solution.

T: Speech comprehension is another fundamental main field. The capacity to be associated with this may exist from birth in its most original form. It is first connected with estimates of area, distance, quantity etc. Different numbers of objects are observed and recognised. Experiences are differentiated. At an early age, reactions appear which relate to "very little", e.g. food. Natural numbers are identified first. Experiences of "two" occur at age 2, if not earlier. Negative numbers, rational numbers and real numbers are comprehended late during adolescence.

Causes of error include, for example, lack of experience of quantities, uncertainty of tables and difficulty in counting objects, deficiencies in the ordering of numbers and vague logic about the use of numbers, numerical quantities and symbols.

G: Form (and body) perception, geometry, measurement etc.: Certain reactions in field G are observed from birth. Field G develops while the child becomes aware of shapes and distinguishes them with the aid of logical thinking, through its own movements, by itself depicting or creating shapes, classifying and giving names to shapes and measuring objects.

The causes of error are above all lack of practical experience, but also undeveloped perception and motor skills and uncertainty with regard to linguistic and other symbols.

It is remarkable that many schoolchildren have great gaps in their knowledge of exactly these primary main fields. Perhaps we underrate the importance of working systematically in these fields.

The three remaining fields can be considered secondary in relation to those we have already mentioned. Perhaps infant children also have a capacity for experiencing probability, using ideas of function and dividing, increasing, reducing, comparing and multiplying. But the curriculum only introduces the idea of "increase" from the commencement of school. Division is introduced far too late. Perhaps indeed division and subtraction have older psychological roots than multiplication and addition. Who has not seen infant children sharing things equally between them? Experience of this kind lays the foundations of division and multiplication.

Concerning ASMD, the four rules of arithmetic, we have found the following: They require speech comprehension, sufficient mathematical language comprehension and a form of perception which can be used in abstract reasoning.

The main causes of error are deficiencies in these respects.

Some applications

Magne's and Thörn's cognitive taxonomy has already been tested. With it, very sharp scrutiny is possible, for example, of teaching materials, curricula and diagnostic materials.

The taxonomy makes it possible

- to diagnose pupils' knowledge of mathematics
- to find methods for pupils' learning
- to construct learning programmes
- to construct curricula/working plans for mathematics
- to write counting aids and other teaching materials
- to analyse teaching materials
- to construct diagnostic tasks for pupils
- to construct marking systems
- to devise computerised aids

- o to facilitate mathematics teaching in pre-school education
- o to facilitate the training of mathematics teachers
- o to develop systems for research purposes.

Magne's (1982) Medelsta study employed a diagnostic method evolved from the Magne-Thörn taxonomy. This study suggests that compulsory school pupils (grades 1-9) have a relatively sound knowledge of the four rules of arithmetic. On the other hand they have serious gaps in the other main fields. Using the cognitive taxonomy, it is possible to make in-depth analyses of the pupils' all-round knowledge.

This result can be explained by another study presented by the same authors, showing that several arithmetic textbooks over-emphasise the four rules of arithmetic and allow little scope, for example, for fields P, T and G. It seems possible that the arithmetic books of the 1930s and 1940s often presented a more balanced structure of subject matter than the books being used today. The scrutiny of mathematics textbooks could also benefit from a theoretical basis of the kind offered by the Magne-Thörn taxonomy.

One also finds that a number of common commercial diagnostic programmes are incomplete in the sense of conveying biased information about the pupils' knowledge. Here again, guidance is obtainable from the cognitive taxonomy.

One of the authors' aims is to achieve a balance between the main fields and methods of learning and teaching.

- o Balance between the main fields: each main field must be given a good chance of being learned.

- o Balance between experience, conceptual teaching and applications: the teacher should use a variety of methods.

- o Balance between forms of organisation: the pupil's learning must proceed in differently sized groups, so as to benefit the dispositions of different pupils.

o Balance between physical teaching materials: neither teachers nor pupils should be heavily dependent on one arithmetic textbook. Verbal and written production must alternate. Varying media should confront the senses. Laboratory methods must be used when needed.

The surveys include analyses, the main outlines of which can be summarised as follows:

o A theory of learning and education for the elementary teaching of mathematics should emanate from a cognitive taxonomy.

o It is proposed that a taxonomy of this kind be based on what is termed the task process model.

o Pedagogical models should emanate from:
(1) Realisation that the learner must be in the focus of school activities.
(2) Balance between learning conditions.
(3) An epistemological approach.

o The cognitive taxonomy can be applied to educational planning in many ways.

o Curricula, study programmes and teaching materials can be based on the cognitive taxonomy.

o Teaching items in mathematics should be extensively redeployed in accordance with the proposed category system.

o The cognitive taxonomy provides a basis for research and development work.

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SCHOOL RESEARCH NEWSLETTER

School Research Newsletter contains reviews of current and terminated research within the field of education.

School Research Newsletter is published and distributed in an edition of 600 issues ten times per year. The Newsletter is obtained free of charge.

Published by Swedish National Board of Education,
Department for Coordination and Planning.
S-106 42 Stockholm. Telephone +46 8 783 20 00
Responsible Editor: Inger Marklund. Editor: Gunilla Karlsson
ISSN 0345-5343